

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
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7	
8	
9	
TOTAL	



General Certificate of Education  
Advanced Level Examination  
June 2011

# Mathematics

# MPC3

## Unit Pure Core 3

Monday 13 June 2011 9.00 am to 10.30 am

**For this paper you must have:**

- the blue AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

**Time allowed**

- 1 hour 30 minutes

- Instructions**
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
  - Fill in the boxes at the top of this page.
  - Answer **all** questions.
  - Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
  - You must answer the questions in the spaces provided. Do not write outside the box around each page.
  - Show all necessary working; otherwise marks for method may be lost.
  - Do all rough work in this book. Cross through any work that you do not want to be marked.

- Information**
- The marks for questions are shown in brackets.
  - The maximum mark for this paper is 75.

- Advice**
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.







**2 (a) (i)** Find  $\frac{dy}{dx}$  when  $y = xe^{2x}$ . (3 marks)

**(ii)** Find an equation of the tangent to the curve  $y = xe^{2x}$  at the point  $(1, e^2)$ . (2 marks)

**(b)** Given that  $y = \frac{2 \sin 3x}{1 + \cos 3x}$ , use the quotient rule to show that

$$\frac{dy}{dx} = \frac{k}{1 + \cos 3x}$$

where  $k$  is an integer. (4 marks)

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- 3** The curve  $y = \cos^{-1}(2x - 1)$  intersects the curve  $y = e^x$  at a single point where  $x = \alpha$ .
- (a)** Show that  $\alpha$  lies between 0.4 and 0.5. (2 marks)
- (b)** Show that the equation  $\cos^{-1}(2x - 1) = e^x$  can be written as  $x = \frac{1}{2} + \frac{1}{2}\cos(e^x)$ . (1 mark)
- (c)** Use the iteration  $x_{n+1} = \frac{1}{2} + \frac{1}{2}\cos(e^{x_n})$  with  $x_1 = 0.4$  to find the values of  $x_2$  and  $x_3$ , giving your answers to three decimal places. (2 marks)

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4 (a) (i) Solve the equation  $\operatorname{cosec} \theta = -4$  for  $0^\circ < \theta < 360^\circ$ , giving your answers to the nearest  $0.1^\circ$ . (2 marks)

(ii) Solve the equation

$$2 \cot^2(2x + 30^\circ) = 2 - 7 \operatorname{cosec}(2x + 30^\circ)$$

for  $0^\circ < x < 180^\circ$ , giving your answers to the nearest  $0.1^\circ$ . (6 marks)

(b) Describe a sequence of two geometrical transformations that maps the graph of  $y = \operatorname{cosec} x$  onto the graph of  $y = \operatorname{cosec}(2x + 30^\circ)$ . (4 marks)

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The functions  $f$  and  $g$  are defined with their respective domains by

$$f(x) = x^2 \quad \text{for all real values of } x$$

$$g(x) = \frac{1}{2x + 1} \quad \text{for real values of } x, \quad x \neq -0.5$$

- (a) Explain why  $f$  does not have an inverse. (1 mark)
- (b) The inverse of  $g$  is  $g^{-1}$ . Find  $g^{-1}(x)$ . (3 marks)
- (c) State the range of  $g^{-1}$ . (1 mark)
- (d) Solve the equation  $fg(x) = g(x)$ . (3 marks)

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6 (a) Given that  $3 \ln x = 4$ , find the exact value of  $x$ . (1 mark)

(b) By forming a quadratic equation in  $\ln x$ , solve  $3 \ln x + \frac{20}{\ln x} = 19$ , giving your answers for  $x$  in an exact form. (5 marks)

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**7 (a)** On separate diagrams:

**(i)** sketch the curve with equation  $y = |3x + 3|$ ; (2 marks)

**(ii)** sketch the curve with equation  $y = |x^2 - 1|$ . (3 marks)

**(b) (i)** Solve the equation  $|3x + 3| = |x^2 - 1|$ . (5 marks)

**(ii)** Hence solve the inequality  $|3x + 3| < |x^2 - 1|$ . (2 marks)

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Use the substitution  $u = 1 + 2 \tan x$  to find

$$\int \frac{1}{(1 + 2 \tan x)^2 \cos^2 x} dx \quad (5 \text{ marks})$$

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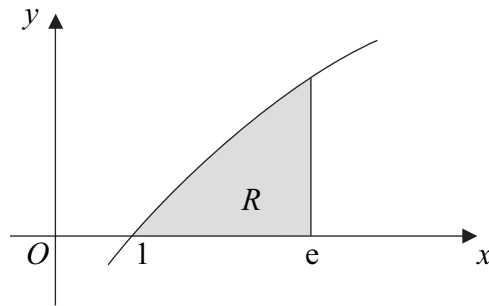




9 (a) Use integration by parts to find  $\int x \ln x \, dx$ . (3 marks)

(b) Given that  $y = (\ln x)^2$ , find  $\frac{dy}{dx}$ . (2 marks)

(c) The diagram shows part of the curve with equation  $y = \sqrt{x} \ln x$ .



The shaded region  $R$  is bounded by the curve  $y = \sqrt{x} \ln x$ , the line  $x = e$  and the  $x$ -axis from  $x = 1$  to  $x = e$ .

Find the volume of the solid generated when the region  $R$  is rotated through  $360^\circ$  about the  $x$ -axis, giving your answer in an exact form. (6 marks)

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A series of horizontal dotted lines for writing the answer.





